Market equilibrium in negotiations and growth models

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THE FOURTH INTERNATIONAL CONFERENCE on GAME THEORY AND MANAGEMENT, St-Petersburg, 28-30 June, 20

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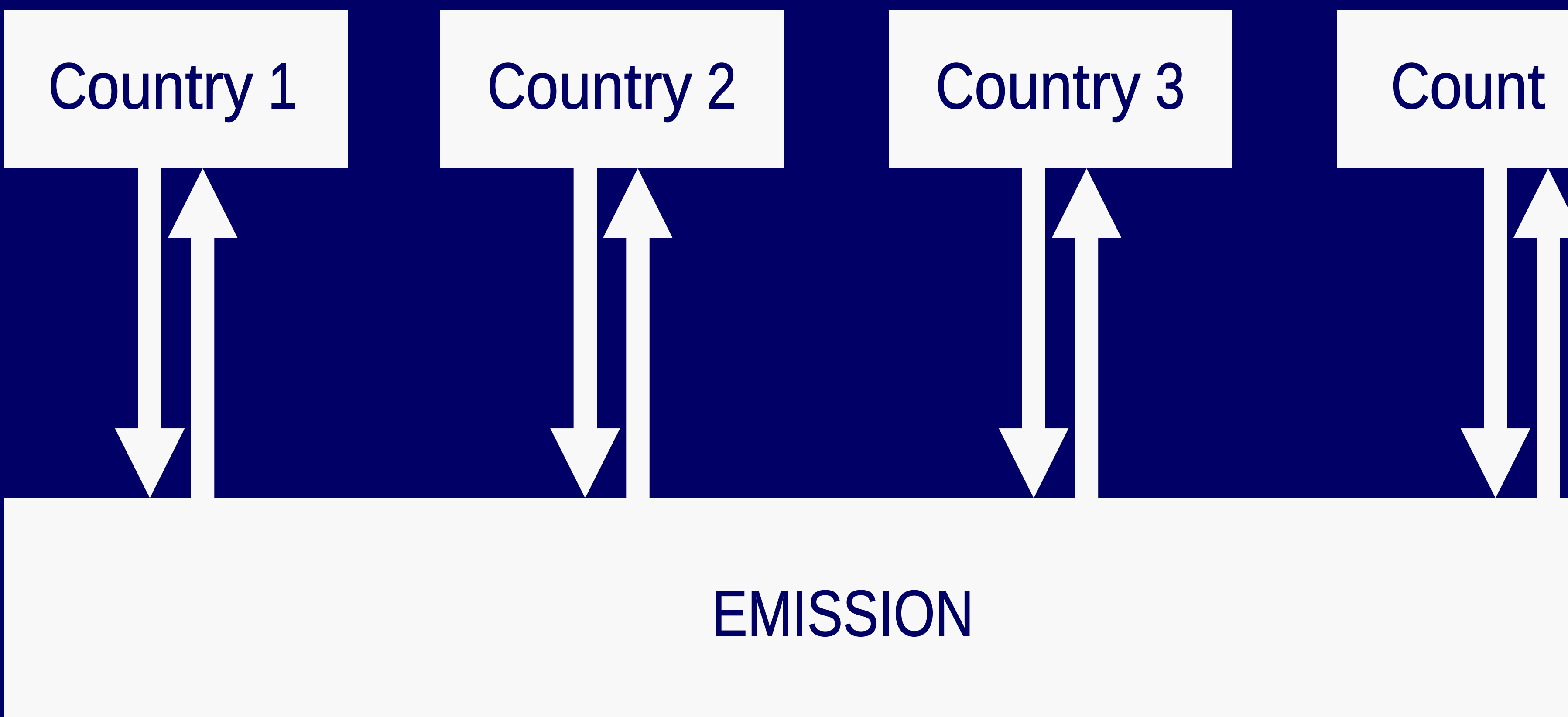
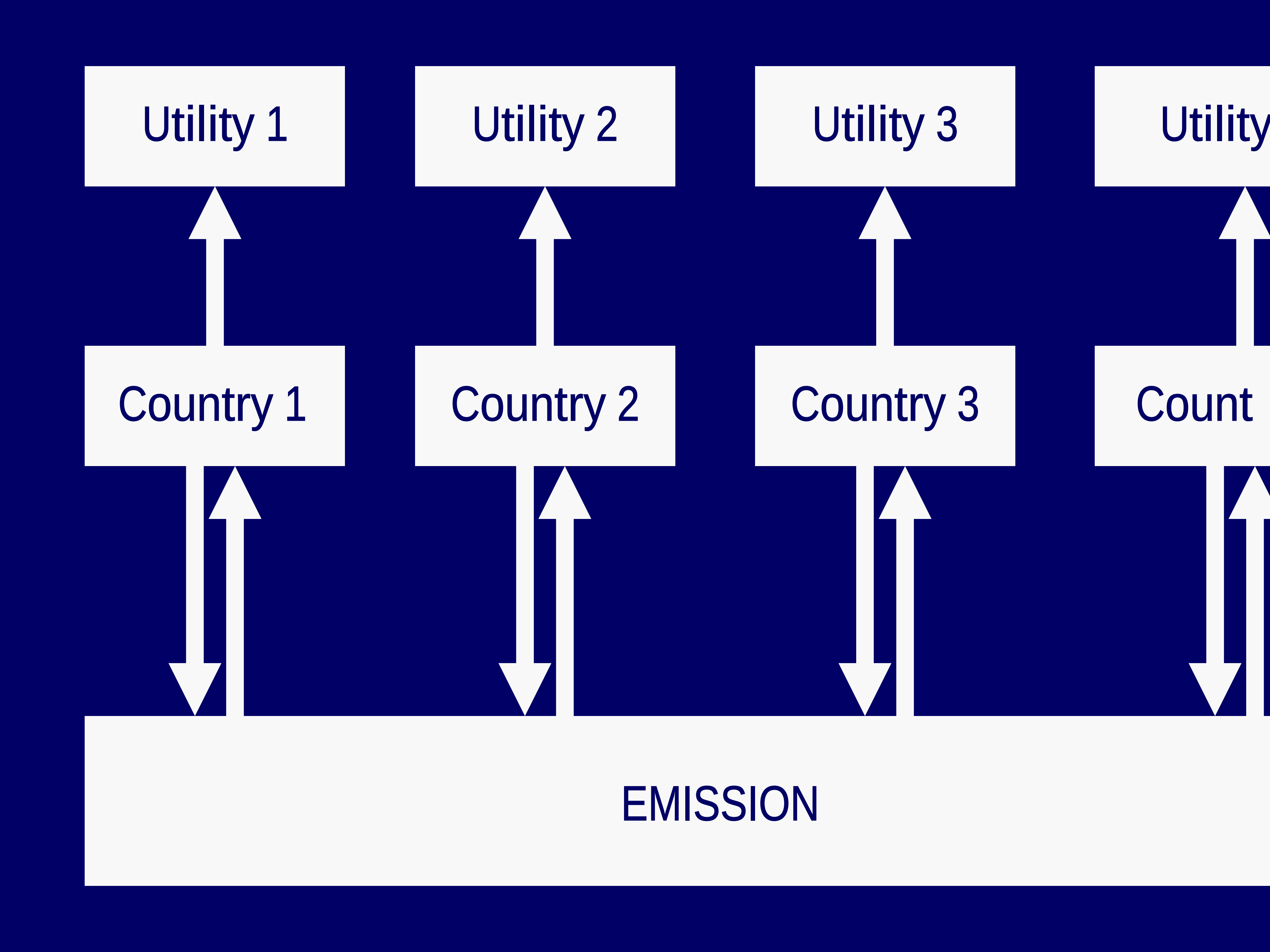

Country 1
Country 2
Country 3
Count
EMISSION


Utility 1
Utility 2
Utility 3
Utility
Country 1
Country 2
Country 3
Count
EMISSION

Country $i \quad (i = 1, \ldots, n)$

Country $i \quad (i = 1, \ldots, n)$

x_i

emission reduction

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x_i	emission reduction
$r_i(x_i)$	cost for x_i

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$b_i(x_1, \ldots, x_n)$	benefit from $x_1, \ldots, x_n$

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$r_i(x_i)$	cost for x_i
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$W_i = b_i - r_i$	utility

Equilibrium

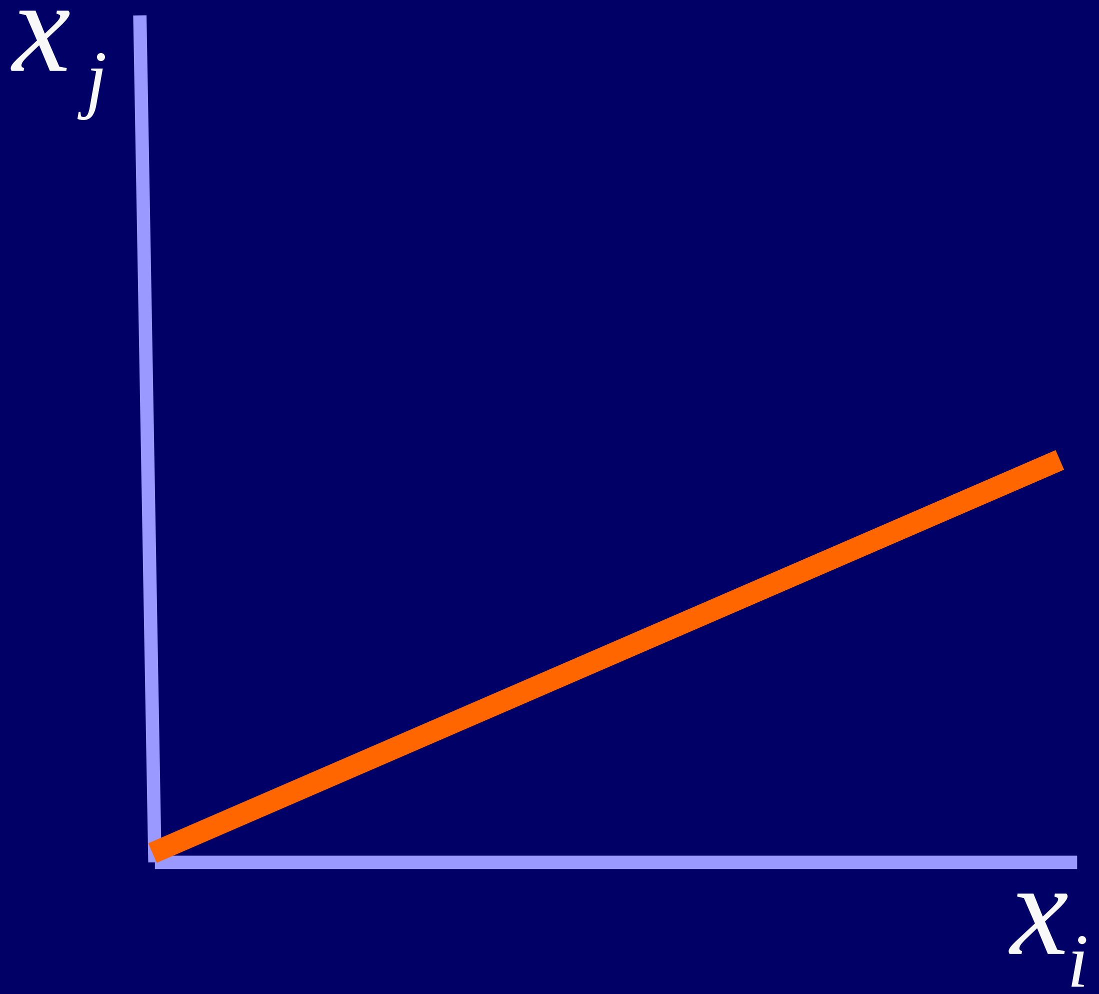
x_i	emission reduction
$r_i(x_i)$	cost for x_i
$b_i(x_1,\ldots,x_n)$	benefit from $x_1,\ldots,x_n$
$W_i = b_i - r_i$	utility
λ_{ij}	i's price for x_j

Equilibrium

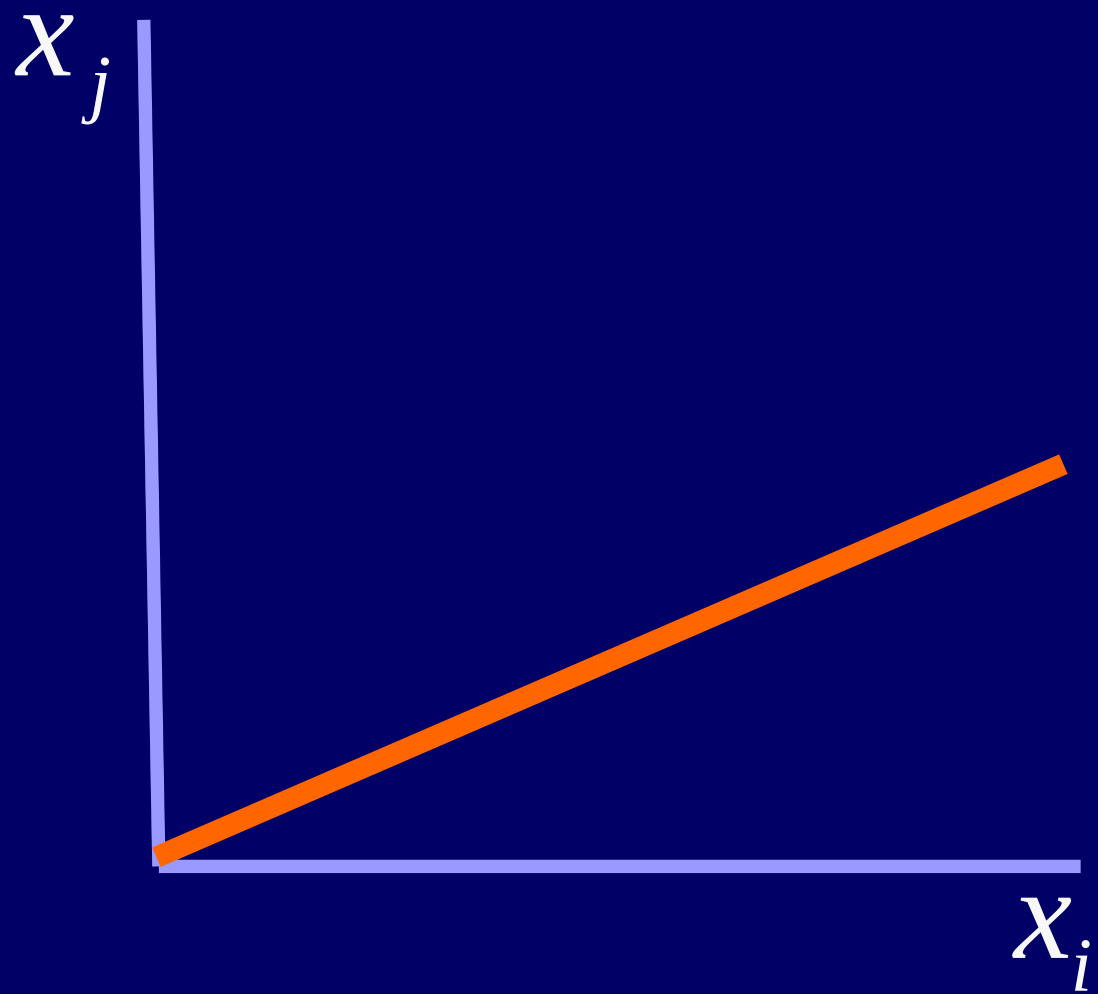
$$x_j = \lambda_{ij} x_i$$

Equilibrium

$$x_j = \lambda_{ij} x_i$$



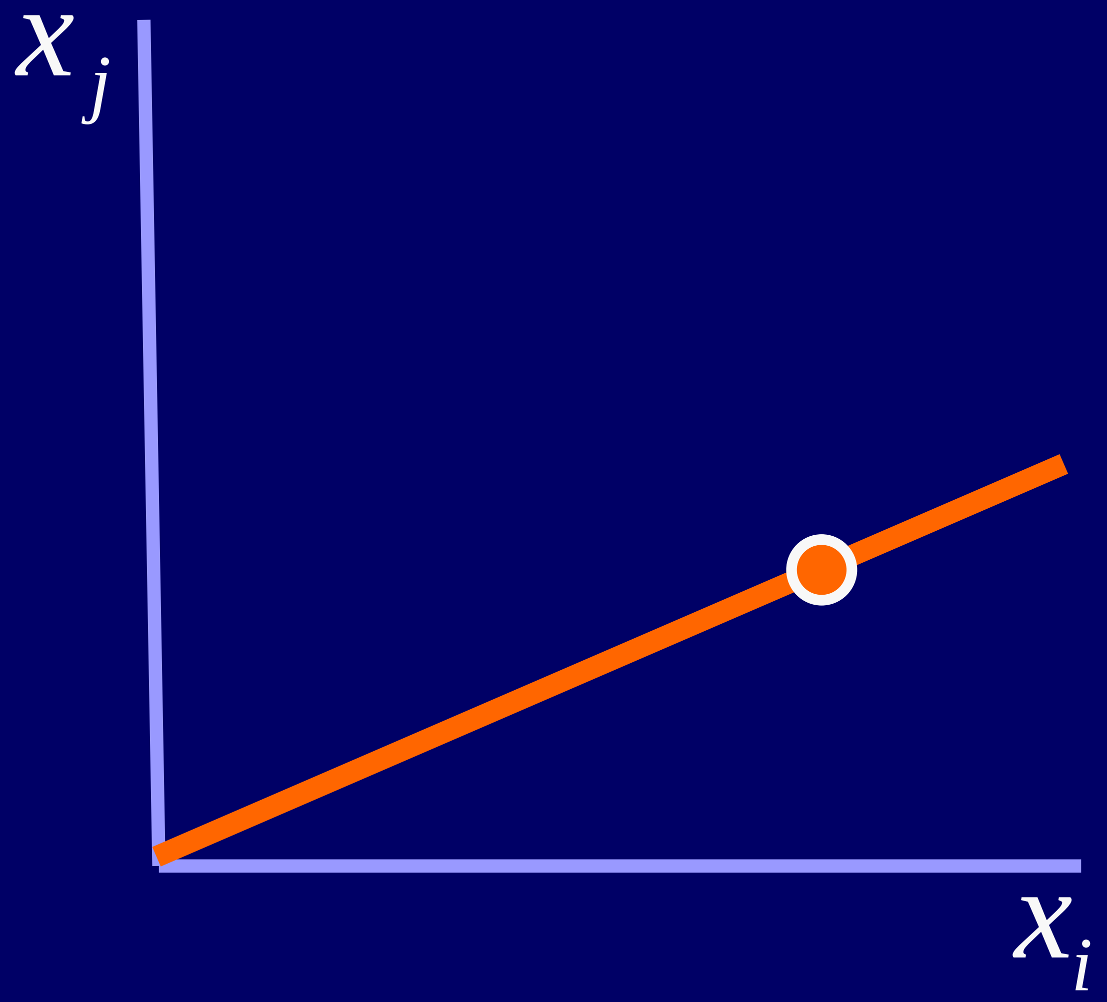
Equilibrium



$$x_j = \lambda_{ij} x_i$$

$$W_i \to \max$$

Equilibrium



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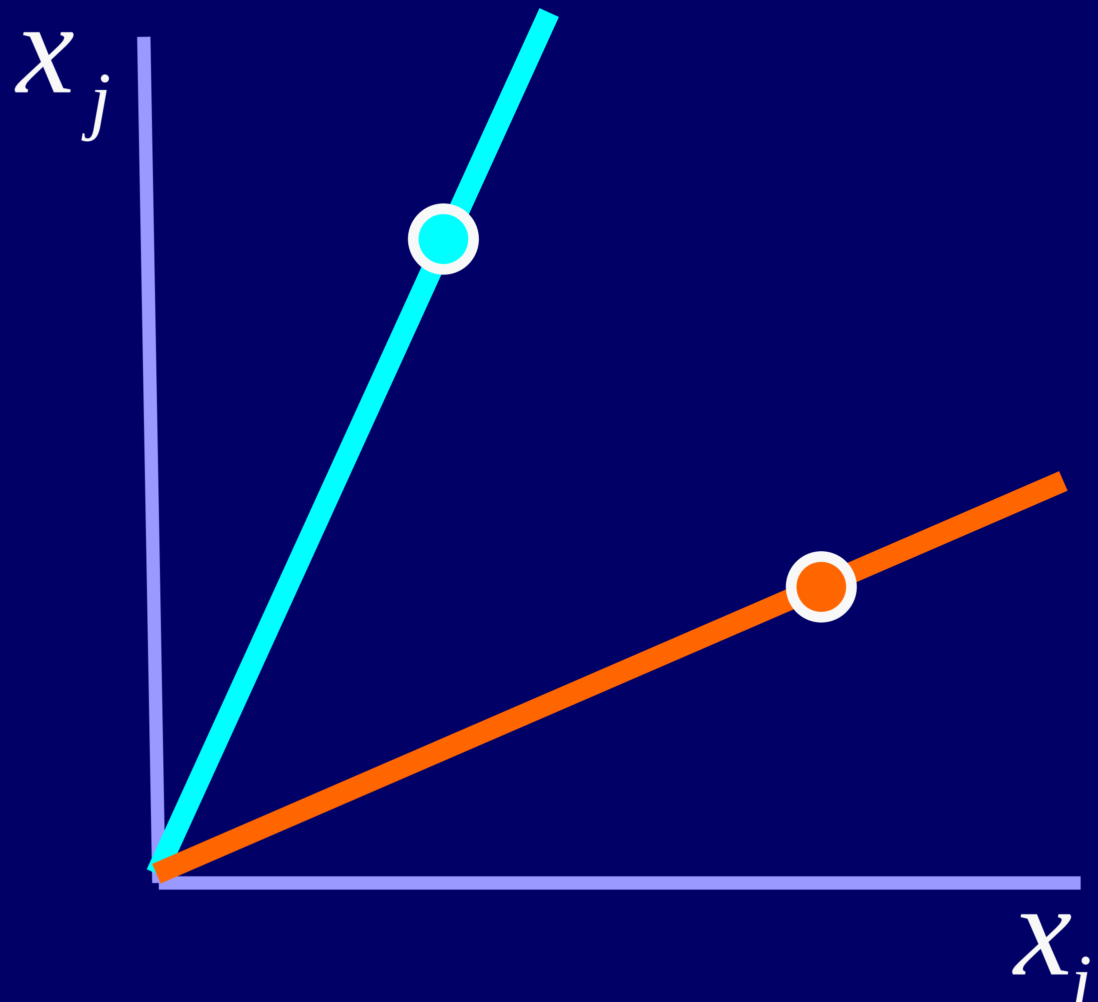

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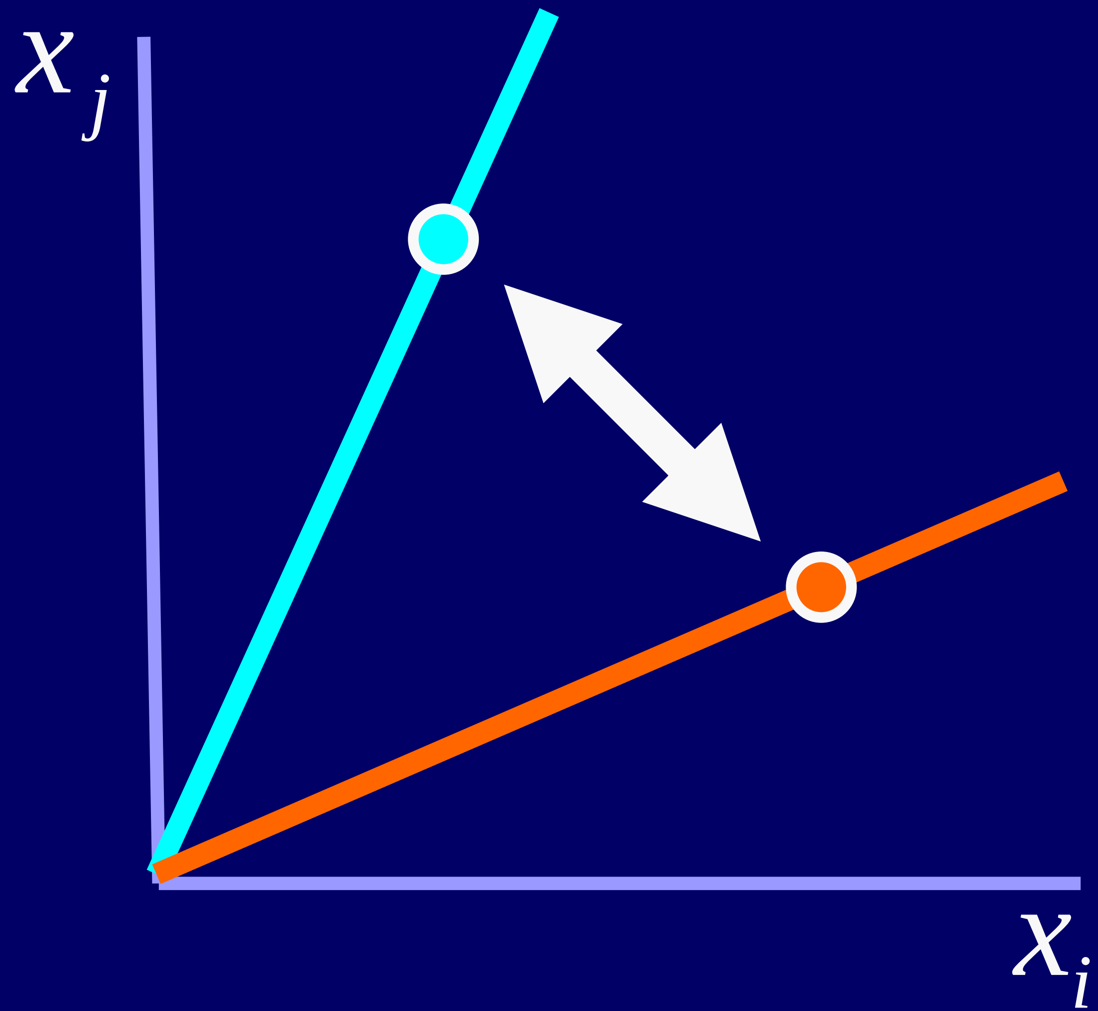

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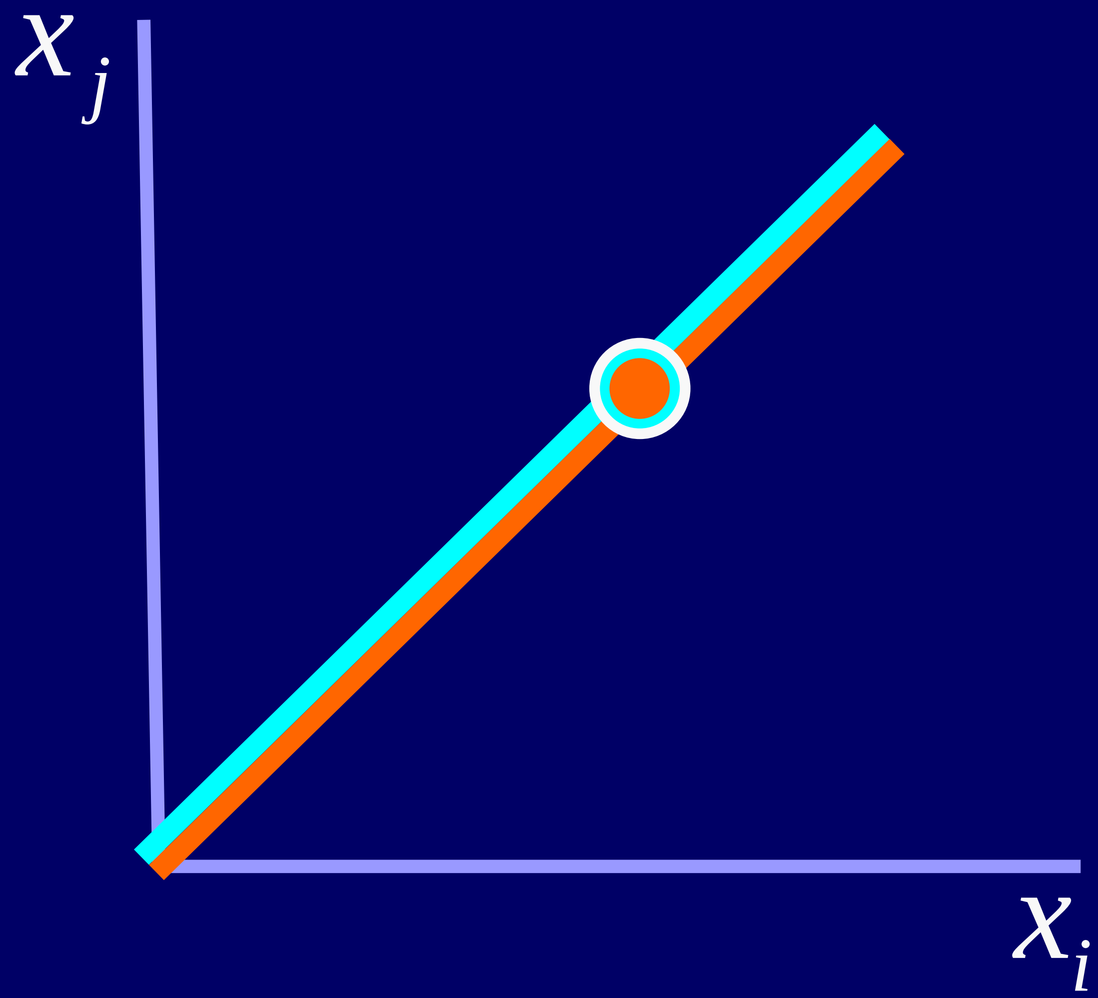

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Equilibrium

$$(\lambda_{ij}) \to (x_1, \ldots, x_n)$$

Equilibrium

$$(\lambda_{ij}) \to (x_1, \ldots, x_n) \to (\lambda_{ij} = x_i / x_j)$$

Market equilibrium
in negotiations and growth models

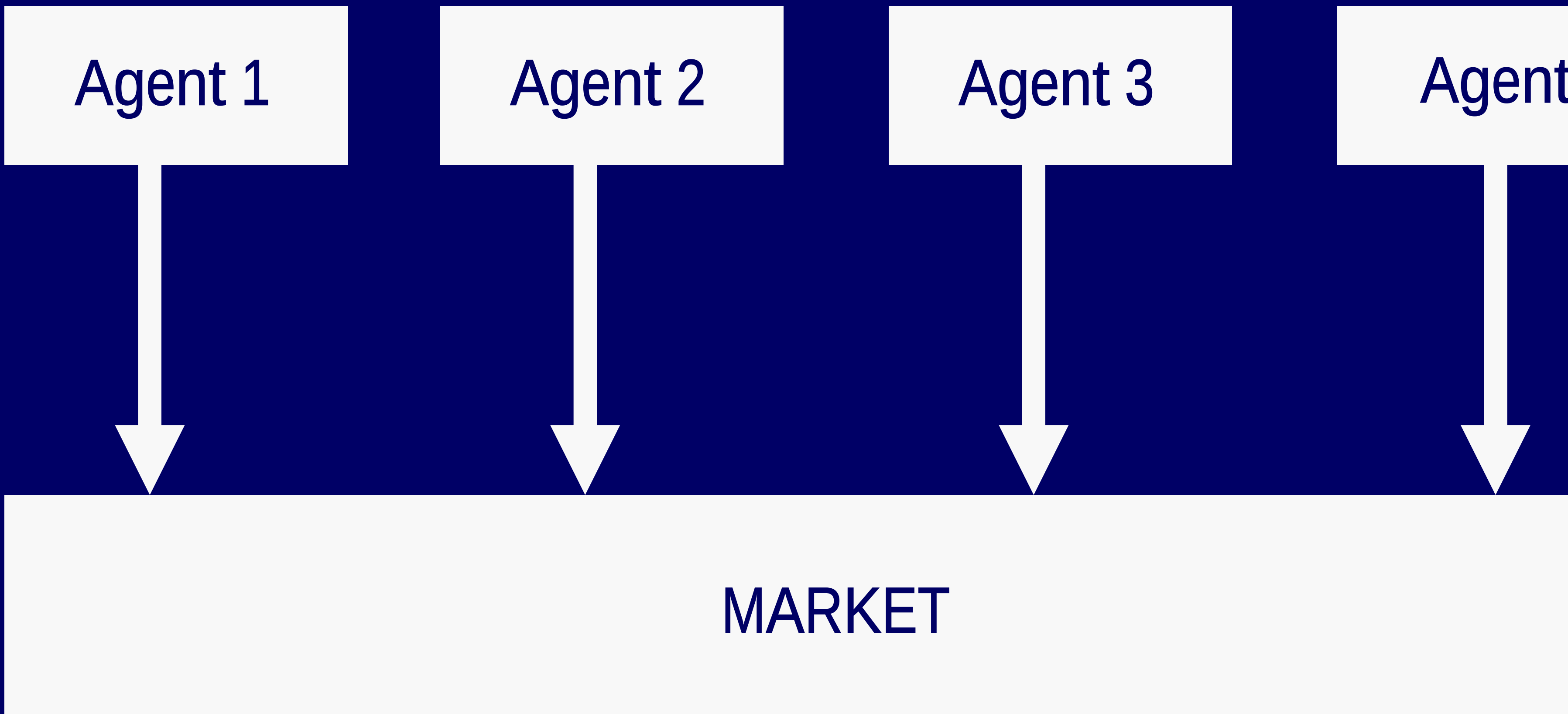
Agent 1

Agent 2

Agent 3

Agent


Agent 1
Agent 2
Agent 3
Agent
MARKET


Agent 1
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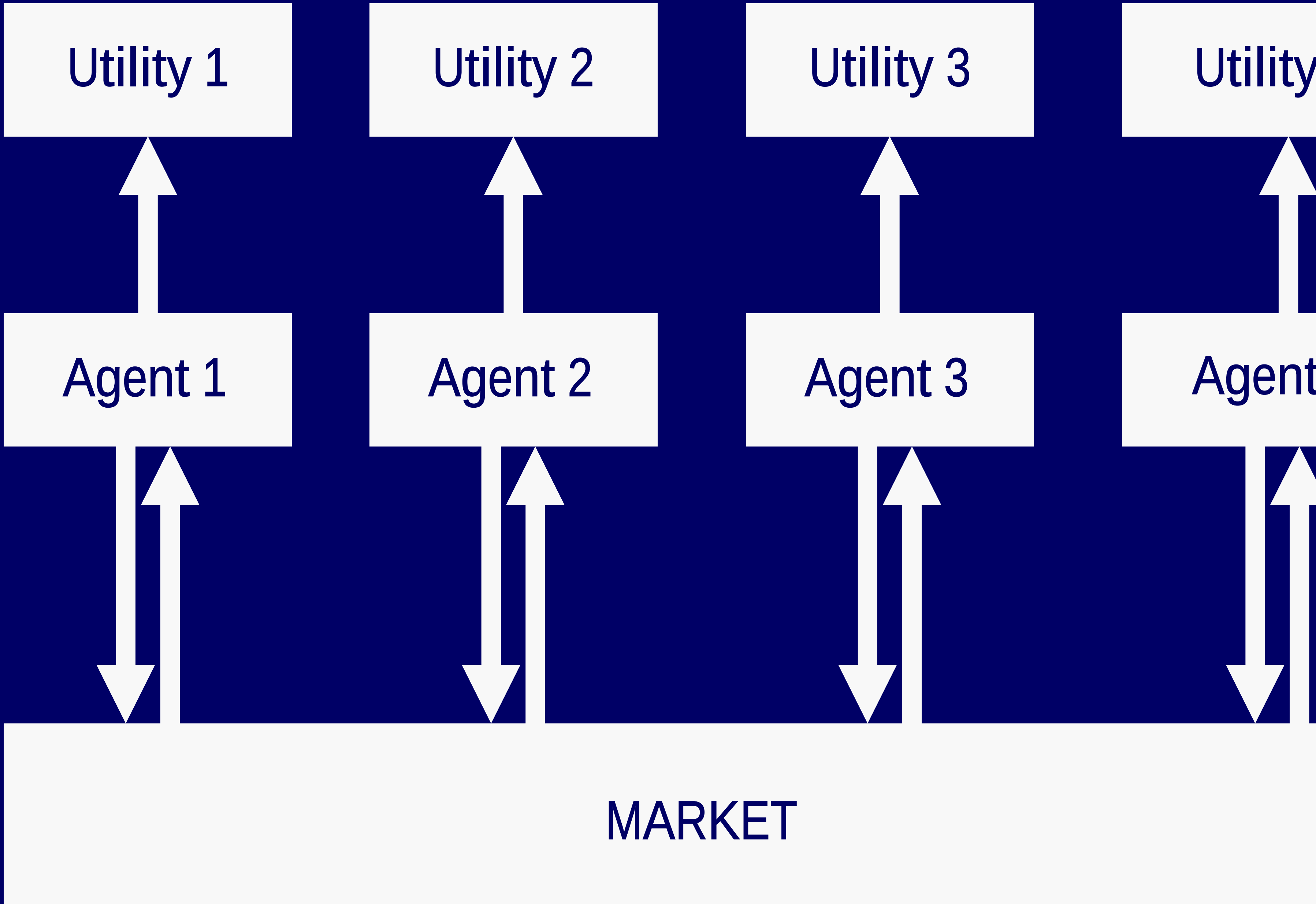

Utility 1
Utility 2
Utility 3
Agent 1
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Agent 3
MARKET

Agent $i \quad (i = 1, \ldots, n)$

Agent $i \quad (i = 1, \ldots, n)$

k_i

capital

Agent $i \quad (i = 1, \ldots, n)$

$$k_i$$

capital

$$y_i = a_i k_i$$

products for market

Agent $i \quad (i = 1, \ldots, n)$

k_i capital

$y_i = a_i k_i$ products for market

p_i price

Agent $i \quad (i = 1, \ldots, n)$

k_i	capital
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c_{ij}	purchased part of y_j

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$C_i = c_{i1}^{\gamma_1} \ldots c_{in}^{\gamma_n}$	consumption

Agent $i \quad (i = 1, \ldots, n)$

$\dot{k}_i =$	capital dynamics
$y_i = a_i k_i$	products for market
p_i	price
c_{ij}	purchased part of
$C_i = c_{i1}^{\gamma_1} \ldots c_{in}^{\gamma_n}$	consumption

Agent $i \quad (i = 1, \dots, n)$

$$\dot{k}_i = u_i k_i - \delta_i k_i + h_i$$

capital dynamics

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$$\dot{k}_i = u_i k_i - \delta_i k_i + h_i$$

capital dynamics

$$u_i \in [0,1]$$

capital saving rate

Agent $i \quad (i = 1, \ldots, n)$

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capital dynamics

$$u_i \in [0,1]$$

capital saving rate

$$h_i = \sum_j p_i c_{ji}$$

income from sales

Agent $i \quad (i = 1, \ldots, n)$

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$$J_i = \int_0^\infty e^{-\rho t} \log C_i \, dt$$

utility

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capital dynamics

$$u_i \in [0,1]$$

capital saving rate

$$(1 - u_i) k_i = \sum_j p_j c_{ij}$$

spending for consumption

Maximize

$$J_i = \int_0^\infty e^{-\rho t} \left(\sum_j \gamma_j \log c_{ij} \right) dt$$

utility

Agent $i \quad (i = 1, \ldots, n)$

$$\dot{k}_i = u_i k_i - \delta_i k_i + h_i$$

capital dynamics

$$u_i \in [0,1]$$

capital saving rate

$$c_{ij} = \frac{\gamma_i (1 - u_i) k_i}{\gamma p_j}$$

$$\gamma = \sum_j \gamma_j$$

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$$J_i = \int_0^\infty e^{-\rho t} [\log k_i + \log(1 - u_i)] dt$$

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$$u_i \in [0,1]$$

$$k_i(0) = k_i^0$$

$$J_i = \int_0^\infty e^{-\rho t} [\log k_i + \log(1 - u_i)] dt$$

Game

$$\dot{k}_i = u_i k_i - \delta_i k_i + \frac{\gamma_i}{\gamma} \sum_j (1 - u_j) k_j$$

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$$\boxed{(1-u_j)k_j}$$

j's spending

Game

$$\dot{k}_i = u_i k_i - \delta_i k_i + \frac{\gamma_i}{\gamma} \sum_j \boxed{(1-u_j)k_j}$$

$$(1-u_j)k_j$$

j's spending

$$y_i = a_i k_i$$

i's products

Game

$$\dot{k}_i = u_i k_i - \delta_i k_i + \frac{\gamma_i}{\gamma} \sum_j \boxed{(1-u_j)k_j}$$

$$(1-u_j)k_j$$

j's spending

$$y_i = a_i k_i$$

i's products

$$\lambda_{ji} = \frac{(1-u_j)k_j}{a_i k_i}$$

j's price for i's products

Game

$$\dot{k}_i = u_i k_i - \delta_i k_i + \frac{\gamma_i}{\gamma} \sum_j \boxed{(1-u_j)k_j}$$

$$\boxed{(1-u_j)k_j = \lambda_{ji} a_i k_i}$$

j's spending

$$y_i = a_i k_i$$

i's products

$$\lambda_{ji} = \frac{(1-u_j)k_j}{a_i k_i}$$

j's price for *i*'s products

Game

$$\dot{k}_i = u_i k_i - \delta_i k_i + \frac{\gamma_i}{\gamma} a_i \lambda_i k_i$$

$$\lambda_{ji} = \frac{(1 - u_j) k_j}{a_i k_i}$$

$$\lambda_i = \sum_j \lambda_{ji}$$

Game

$$\dot{k}_i = u_i k_i - \delta_i k_i + \frac{\gamma_i}{\gamma} a_i \lambda_i k_i$$

$$u_i \in [0,1]$$

$$k_i(0) = k_i^0$$

$$J_i = \int_0^\infty e^{-\rho t}[\log k_i + \log(1-u_i)]dt \longrightarrow \max$$

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$$\lambda_i = \sum_j \lambda_{ji}$$

Game

$$\dot{k}_i = u_i k_i - \delta_i k_i + \frac{\gamma_i}{\gamma} a_i \lambda_i k_i$$

$$u_i = 1 - \rho$$

$$k_i(0) = k_i^0$$

$$J_i = \int_0^\infty e^{-\rho t}[\log k_i + \log(1-u_i)]dt \rightarrow \max$$

$$\lambda_{ji} = \frac{(1-u_j)k_j}{a_i k_i}$$

$$\lambda_i = \sum_j \lambda_{ji}$$

Game

$$\dot{k}_i = u_i k_i - \delta_i k_i + \frac{\gamma_i}{\gamma} a_i \lambda_i k_i$$

$$u_i = 1 - \rho$$ robust to $$a_j, p_j, \lambda_{ji}, k_j^0, \gamma_j$$

$$J_i = \int_0 e^{-\rho t}[\log k_i + \log(1-u_i)]dt \longrightarrow \max$$

$$k_i(0) = k_i^0$$

$$\lambda_{ji} = \frac{(1-u_j)k_j}{a_i k_i}$$

$$\lambda_i = \sum_j \lambda_{ji}$$

Game

$$\dot{k}_i = \boxed{u_i} k_i - \delta_i k_i + \frac{\gamma_i}{\gamma} a_i \lambda_i k_i$$

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$u_i = 1 - \rho$ robust to $a_j, p_j, \lambda_{ji}, k_j^0, \gamma_j$

k_i, λ_{ji} robust to p_j, γ_j

$$\lambda_{ji} = \frac{(1-u_j)k_j}{a_i k_i}$$

$$\lambda_i = \sum_j \lambda_{ji}$$

Constraints

$$\sum_j c_{ji} \leq y_i$$

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$$\rho = 1 - u_i$$

$$c_{ji} = \frac{\gamma_j \rho k_j}{\gamma p_i}$$

$$y_i = a_i k_i$$

Constraints

$$\sum_j c_{ji} \leq y_i$$

$$\rho = 1 - u_i$$

$$c_{ji} = \frac{\gamma_j \rho k_j}{\gamma p_i}$$

$$y_i = a_i k_i$$

$$p_i \geq \frac{\rho}{a_i} \sum_j \frac{\gamma_j}{\gamma} \frac{k_j}{k_i}$$

Further steps

Demand-supply analysis

Equilibrium prices

Open-loop Nash equilibrium

Closed-loop Nash equilibrium

Pareto equilibrium

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